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Generation of two-flavor vortex atom laser from a five-state medium

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Abstract. Two-flavor atom laser in a vortex state is obtained via electromagnetically induced transparency (EIT) technique in a five-level M type system by using two probe lights with $\pm z$-directional orbital angular momentum $\pm l\hbar$, respectively. Together with the original transfer technique of quantum states from light to matter waves, the present result can be extended to generate continuous two-flavor vortex atom laser with non-classical atoms.

PACS. 03.75.-b Matter waves – 42.50.Gy Effects of atomic coherence on propagation, absorption, and amplification of light; electromagnetically induced transparency and absorption – 03.65.Ta Foundations of quantum mechanics; measurement theory

1 Introduction

Since the remarkable observations of Bose-Einstein Condensation (BEC) in dilute atomic clouds in 1995, there have been many interests in preparing a continuous atom laser [1] and exploring its potential applications in, e.g., gravity measurements through atom interferometry [2]. Although a sub-quantum-noise atom laser is expected to be crucial to improve the interferometer sensitivities, the difficulties for the atomic beam to propagate over a long distance heavily restrict its actual performance [3]. Some time ago Drummond et al. proposed to use mode-locking technique to stabilize the atom laser based on the generation of a dark soliton in a ring-shaped condensate [4]. Other related works are the atomic soliton formation and its stationary transmission in a traveling optical laser beam [5] or a waveguide [6] for a dense atomic flow. Also, an optical scheme for generating the soliton atom laser via electromagnetically induced transparency (EIT) [7] was proposed in our recent work [8].

The generation of a vortex condensate with a Raman adiabatic passage scheme has been early proposed in the three-level system [9]. A proposal to create a skyrmion in a Bose-Einstein Condensate with an $F = 1$ hyperfine ground state manifold (three ground states) using Raman transitions has been made by Marzlin et al. [10], and the importance of a tight external trap for a high efficiency transfer in such a system was analyzed in detail [11]. In these techniques a sufficiently large one-photon detuning was needed to avoid the spontaneous emission from the excited state. Recently, the exchange of orbital momentum between the light and molecules was also studied by Babiker et al. [12]. On the other hand, Juzeliunas and Öhberg probed the influence of slow light with an orbital momentum angular on the mechanical properties of three-level atomic Degenerate Fermi Gases (DFG) and pointed out the existence of an effective magnetic field and its analogy de Haas-van Alphen effect in the neutral DFG system [13]. For the three-level case, to obtain the stable vortex state, one usually requires that the ratio of Rabi-frequencies between probe and control fields should be (approximately) independent of space [9,11]. In this Letter, we proceed to generate two-flavor atom laser in vortex states by extending the Juzeliunas-Ohberg model to a five-level M type atomic system. The equations of motion for the output atomic beams can be described as that of an effective two-flavor (oppositely charged) Bose condensate in an effective magnetic field, and the ratios of Rabi-frequencies in this technique are not needed to be constant.

The present technique was based on the physical mechanism of Electromagnetically Induced Transparency (EIT) [7], which has attracted much attention in both experimental and theoretical aspects in recent years [14–16], especially after Fleischhauer and Lukin formulated their elegant dark-states polaritons (DSPs) theory [17] and

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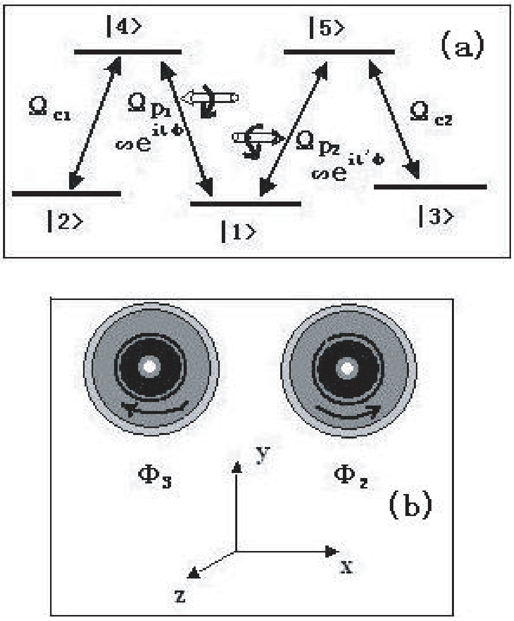

Fig. 1. (a) The condensate atoms with internal five-level M type configuration interact with two probe and two control beams, where the two probe beams have orbital angular momentum $\hbar l$ and $\hbar l'$, respectively in the z direction. (b) Output two-flavor atom lasers in corresponding vortex states (in $\pm z$ vortical direction).

thereby the rapid developments of quantum memory technique, i.e., transferring the quantum states of photon wave-packet to collective Raman excitations in a loss-free and reversible manner. By extending the transfer technique to matter waves, Fleischhauer and Gong proposed a wonderful scheme to make a continuous atomic beam with non-classical or entangled states [18], which was later confirmed for double-Λ four-level atomic medium [20], even to a soliton atom laser [8].

In the following we propose a technique to generate two-flavor atom lasers in vortex states via induced Raman transitions in a five-level M type system which interacts with two probe lights which have $\pm z$-directional orbital angular momentum $\pm l\hbar$, respectively, and two external control fields. If the ratio between the Rabi frequencies of the first pair of probe and control fields equals that of the second pair, a vortex state of two-flavor atom laser will be obtained. Together with the original Fleischhauer-Gong scheme [18], the present result can be extended to generate continuous two-flavor vortex atom laser with non-classical atoms. Besides the applications to quantum information, the present system with effective $\pm$ charge may have very interesting applications to spintronics [25].

2 Model

The system we consider is shown in Figure 1a [19]. The condensate atoms with internal five-level M type configuration interact with four laser beams: Two strong control lasers respectively drive the transition $|2\rangle \rightarrow |4\rangle$ with Rabi frequency $\Omega_{c_1} = \Omega_{c_1}^{(0)} \exp(i\mathbf{k}_{c_1} \cdot \mathbf{r})$ and $|3\rangle \rightarrow |5\rangle$ with $\Omega_{c_2} = \Omega_{c_2}^{(0)} \exp(i\mathbf{k}_{c_2} \cdot \mathbf{r})$, where $\Omega_{c_j}^{(0)} (j = 1, 2)$ are the slowly varying amplitudes and k_{c_j} the wave-vectors. The two probe fields coupling the transitions $|1\rangle \rightarrow |4\rangle$ and $|1\rangle \rightarrow |5\rangle$, respectively, are characterized by the wave-vectors $\mathbf{k}_{p_j} = k_{p_j}\hat{\mathbf{z}}(j = 1, 2)$ and the Rabi frequency $\Omega_{p_1} = \Omega_{p_1}^{(0)} e^{i(l\phi + \mathbf{k}_{p_1} \cdot \mathbf{r})}$ and $\Omega_{p_2} = \Omega_{p_2}^{(0)} e^{i(l'\phi + \mathbf{k}_{p_1} \cdot \mathbf{r})}$ where $\Omega_{p_j}^{(0)}$ are also the slowly varying amplitudes. Here l and l' indicate that the probe fields are assumed to respectively have orbital angular momentums $\hbar l$ and $\hbar l'$ along the z-direction. It is convenient to introduce the time-slowly-varying amplitudes $\Psi_2 = \Phi_2 e^{-i(\omega_{p_1} - \omega_{c_1})t}$, $\Psi_4 = \Phi_4 e^{-i\omega_{p_1} t}$, $\Psi_3 = \Phi_3 e^{-i(\omega_{p_2} - \omega_{c_2})t}$, and $\Psi_5 = \Phi_5 e^{-i\omega_{p_2} t}$. Hence the equations of motion for the matter fields are given by

$$i\hbar\frac{\partial\Phi_1}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\Phi_1 + V_1(\mathbf{r})\Phi_1 + \Big[U_{11}|\Phi_1|^2 + U_{12}|\Phi_2|^2 + U_{13}|\Phi_3|^2\Big]\Phi_1 + \hbar\Omega_{p_1}^*\Psi_4 + \hbar\Omega_{p_2}^*\Phi_5, \quad (1)$$

$$i\hbar\frac{\partial\Phi_2}{\partial t} = \left(\epsilon_{12} - \frac{\hbar^2}{2m}\nabla^2\right)\Phi_2 + V_2(\mathbf{r})\Phi_2 + [U_{21}|\Phi_1|^2 + U_{22}|\Phi_2|^2 + U_{32}|\Phi_3|^2]\Phi_1 + \hbar\Omega_{c_1}^*\Phi_4, \quad (2)$$

$$i\hbar\frac{\partial\Phi_3}{\partial t} = \left(\epsilon_{13} - \frac{\hbar^2}{2m}\nabla^2\right)\Phi_3 + V_3(\mathbf{r})\Phi_3 + [U_{31}|\Phi_1|^2 + U_{32}|\Phi_2|^2 + U_{33}|\Phi_3|^2]\Phi_1 + \hbar\Omega_{c_2}^*\Phi_5, \quad (3)$$

$$i\hbar\frac{\partial\Phi_4}{\partial t} = \left(\epsilon_{14} - \frac{\hbar^2}{2m}\nabla^2\right)\Phi_4 + V_4(\mathbf{r})\Phi_4 + \hbar\Omega_{c_1}\Phi_2 + \hbar\Omega_{p_1}\Phi_1, \quad (4)$$

$$i\hbar\frac{\partial\Phi_5}{\partial t} = \left(\epsilon_{15} - \frac{\hbar^2}{2m}\nabla^2\right)\Phi_5 + V_5(\mathbf{r})\Phi_5 + \hbar\Omega_{c_2}\Phi_3 + \hbar\Omega_{p_2}\Phi_1 \quad (5)$$

where $V_i(\mathbf{r})(i = 1, 2, 3, 4, 5)$ are the external potentials, the scattering length a_{ij} characterizes the atom-atom interactions via $U_{ij} = 4\pi\hbar^2 a_{ij}/m$ of which for simplicity we assume the scattering length $a_{ij} = a_0$ is constant. $\epsilon_{14} = \hbar(\omega_{41} - \omega_{p_1})$ and $\epsilon_{15} = \hbar(\omega_{51} - \omega_{p_2})$ are energies of single-photon detunings, while $\epsilon_{12} = \hbar(\omega_{21} - \omega_{p_1} - \omega_{s_1})$ and $\epsilon_{13} = \hbar(\omega_{31} - \omega_{p_2} - \omega_{s_2})$ are energies of the two-photon detunings. Since almost no atoms occupy the excited state $|3\rangle$ in the dark-state condition that was fulfilled in EIT technique, the collisions between two excited states and lower states can safely be neglected.

Assuming that the two-photon detunings ϵ_{12} and ϵ_{13} are sufficiently small and the strengths of the probe fields are much smaller than that of the control fields, one arrives at the adiabatic condition relating Φ_2 and Φ_3 to Φ_1 [13]. Hence, for the dark-state condition [19,21], from equation (4) and (5) we have in the zero order

$$\Phi_2 = -\xi_1\Phi_1, \quad \Phi_3 = -\xi_2\Phi_1, \quad (6)$$

where the ratio coefficients $\xi_1 = -\Omega_{p_1}/\Omega_{c_1}$ and $\xi_2 = -\Omega_{p_2}/\Omega_{c_2}$. With above relations one can easily derive the equations of motion for Φ_1, Φ_2 and Φ_3 [13]. For example, substituting the above formula into the equations (2) and (3), respectively, yields

$$\Phi_4 = -\Big(\hbar\Omega^*_{c_1}\Big)^{-1}\Bigg\{\Big[i\hbar\frac{\partial}{\partial t} - \Big(\epsilon_{12} - \frac{\hbar^2}{2m}\nabla^2\Big) + V_2(\mathbf{r})\Big]\xi_1^{-1}\Phi_1 - \Big(U_{21}|\Phi_1|^2 + U_{22}|\xi_1|^{-2}|\Phi_1|^2 + U_{32}|\xi_2|^{-2}|\Phi_1|^2\Big)\Bigg\}\Phi_1, \quad (7)$$

$$\Phi_5 = -\Big(\hbar\Omega^*_{c_2}\Big)^{-1}\Bigg\{\Big[i\hbar\frac{\partial}{\partial t} - \Big(\epsilon_{13} - \frac{\hbar^2}{2m}\nabla^2\Big) + V_3(\mathbf{r})\Big]\xi_2^{-1}\Phi_1 + \Big(U_{31}|\Phi_1|^2 + U_{32}|\xi_1|^{-2}|\Phi_1|^2 + U_{33}|\xi_2|^{-2}|\Phi_1|^2\Big)\Bigg\}\Phi_1. \quad (8)$$

Together with the above two relations, from equation (1) we can obtain the equation of motion for the field Φ_1. Similarly, from equations (1) and (3) (or Eq. (2)) one can derive the relation between field Φ_4 (Φ_5) and Φ_2 (or Φ_3), and then substitute it to equation (2) (Eq. (3)) we can obtain the equation of motion for the field Φ_2 (or Φ_3). For this we finally find the equations of motion for the matter fields Φ_1, Φ_2 and Φ_3 read

$$i\hbar\frac{\partial}{\partial t}\Phi_\alpha = \frac{1}{2m}\Big[i\hbar\nabla + \vec{A}_\alpha\Big]^2\Phi_\alpha + V_{\alpha eff}\Phi_\alpha + U|\Phi_1|^2\Phi_\alpha, \quad (\alpha = 1,2,3) \quad (9)$$

where we have ignored nonlinear terms involving Φ_2 or Φ_3 for small $|\xi_j|$, the nonlinear interaction strength $U = 4\pi\hbar^2 a_0/m$ and the effective vectors

$$\vec{A}_1 = \hbar\,\Xi_1^{-1}(\xi_1^*\nabla\xi_1 + \xi_2^*\nabla\xi_2), \quad (10)$$

$$\vec{A}_2 = \hbar\,\Xi_2^{-1}\big(1/\xi_2^*\nabla(1/\xi_2) + \xi_1^*/\xi_2^*\nabla(\xi_1/\xi_2)\big), \quad (11)$$

$$\vec{A}_3 = \hbar\,\Xi_3^{-1}\big(1/\xi_1^*\nabla(1/\xi_1) + \xi_2^*/\xi_1^*\nabla(\xi_2/\xi_1)\big), \quad (12)$$

where

$$\Xi_1 = 1 + |\xi_1|^2 + |\xi_2|^2,$$
$$\Xi_2 = 1 + 1/|\xi_2|^2 + |\xi_1|^2/|\xi_2|^2$$
$$\text{and} \quad \Xi_3 = 1 + 1/|\xi_1|^2 + |\xi_2|^2/|\xi_1|^2,$$

and the effective trap potentials

$$V_{1eff} = i\hbar\,\Xi_1^{-1}[V_1 + |\xi_1|^2 V_2 + |\xi_2|^2 V_3 + (2m\Xi_1)^{-1}|\vec{A}_1|^2],$$
$$V_{2eff} = i\hbar\,\Xi_2^{-1}\big[V_2 + (\epsilon_{21} + V_1)/|\xi_2|^2 - V_3|\xi_1|^2/|\xi_2|^2 + (2m\Xi_2)^{-1}|\vec{A}_2|^2\big]$$

and

$$V_{3eff} = i\hbar\,\Xi_3^{-1}\big[V_3 + (\epsilon_{31} + V_1)/|\xi_1|^2 - V_2|\xi_2|^2/|\xi_1|^2 + (2m\Xi_3)^{-1}|\vec{A}_3|^2\big].$$

One can find that the effective vectors $\vec{A}_\alpha (\alpha = 1,2,3)$ is generally non-Hermitian. The Hermitian contribution is due to the changes in the phase of $\xi_{1,2}$. and the non-Hermitian part results in the changes of the amplitude of the bosonic fields Φ_α. Noting that the dimensionless function $\xi_j = e^{iR_j}\Omega^{(0)}_{p_j}/\Omega^{(0)}_{c_j} (j = 1,2)$ where the phases $R_1 = (\mathbf{k}_{p_1} - \mathbf{k}_{c_1})\cdot\mathbf{r} + l\phi$ and $R_2 = (\mathbf{k}_{p_2} - \mathbf{k}_{c_2})\cdot\mathbf{r} + l'\phi$, and under the condition $\big|\nabla|\xi_j|^2\big| \ll \big||\xi_j|^2\nabla R_j\big|$ [13], one can neglect the non-Hermitian part. The effective vectors then yield

$$\vec{A}_1 = \hbar\,\Xi_1^{-1}\Big(|\xi_1|^2\nabla R_1 + |\xi_2|^2\nabla R_2\Big), \quad (13)$$

$$\vec{A}_2 = -\hbar\,\Xi_2^{-1}\Big(|1/\xi_2|^2\nabla R_2 - |\xi_1|^2/|\xi_2|^2\nabla(R_1 - R_2)\Big), \quad (14)$$

$$\vec{A}_3 = \hbar\,\Xi_3^{-1}\Big(|1/\xi_1|^2\nabla R_1 - |\xi_2|^2/|\xi_1|^2\nabla(R_2 - R_1)\Big). \quad (15)$$

When $|\xi_1|^2 = |\xi_2|^2$, i.e., the ratio between the Rabi frequencies $\Omega^{(0)}_{p1}$ and $\Omega^{(0)}_{c1}$ equals that between $\Omega^{(0)}_{p2}$ and $\Omega^{(0)}_{c2}$ (However, one has to keep in mind that the ratio function itself is never restricted), and the photons of two input probe fields have opposite orbital angular momentum, i.e. $l = -l'$, one can easily obtain $\vec{A}_1 = 0$ and $\vec{A}_2 = -\vec{A}_3 = \vec{A} = -\hbar l\nabla\phi$. The equations of (9) can then be rewritten as

$$i\hbar\frac{\partial}{\partial t}\begin{pmatrix}\Phi_2\\ \Phi_3\end{pmatrix} = \frac{1}{2m}\big[i\hbar\nabla + q\sigma_3\vec{A}\big]^2\begin{pmatrix}\Phi_2\\ \Phi_3\end{pmatrix} + \begin{bmatrix}V_{2eff} & 0\\ 0 & V_{3eff}\end{bmatrix}\begin{pmatrix}\Phi_2\\ \Phi_3\end{pmatrix} + U\rho(\mathbf{r})\begin{pmatrix}\Phi_2\\ \Phi_3\end{pmatrix}, \quad (16)$$

where σ_3 is the Pauli matrix, $q = +1$, and

$$i\hbar\frac{\partial}{\partial t}\Phi_1 = -\frac{1}{2m}\hbar\nabla^2\Phi_1 + V_{1eff}(\mathbf{r})\Phi_1. \quad (17)$$

The above equations can easily be understood that the three categories of condensates with effective electric charges $q_1 = 0$ and $q_2 = -q_3 = q = +1$, respectively, interact with an effective external magnetic field $\mathbf{B}_{eff} = \nabla\times\vec{A}$. In other words, by using two probe lights that respectively have orbital angular momentum $\pm l\hbar$ in the $\pm z$-direction, we obtain an effective two-flavor (oppositely charged) condensate which attracts special attention in recent years [22]. For present purpose, we consider the case $|\xi_j|^2 \ll 1$, and have neglected the depletion of atoms in the trapped state $|1\rangle$, and linearized the equation (16) by using [23] $|\Phi_1| \approx \sqrt{\rho(\mathbf{r})}$ with $\rho(\mathbf{r})$ the total density of the condensate. As it is well-known, the ideal configuration for an atom laser can be described as: using a confining magnetic potential to act as a cavity for the laser mode, which is occupied by a Bose-Einstein condensate, radio-frequency or optical Raman transitions are then used to coherently transferred atoms into untrapped hyperfine states, which can propagate freely away from the remaining trapped atoms [24]. For this we may choose $V_2(\mathbf{r})$ and $V_3(\mathbf{r})$ such that $V_{2eff}(\mathbf{r}) = 0$ and $V_{3eff}(\mathbf{r}) = 0$.

Together with equation (6), one can find the solution of equation (16) in the following form

$$\Phi_2(\mathbf{r},t) = -\frac{\Omega_{p_1}^{(0)}}{\Omega_{c_1}^{(0)}}\sqrt{\rho(\mathbf{r})}\exp\left(iS_2(\mathbf{r},t)\right),$$
$$\Phi_3(\mathbf{r},t) = -\frac{\Omega_{p_2}^{(0)}}{\Omega_{c_2}^{(0)}}\sqrt{\rho(\mathbf{r})}\exp\left(iS_3(\mathbf{r},t)\right), \qquad (18)$$

where the phases

$$S_2(\mathbf{r},t) = (q_2/\hbar)\int_0^{\mathbf{r}} \vec{A}\cdot d\mathbf{r}' + S_{02}(\mathbf{r},t)$$
$$\text{and} \quad S_3(\mathbf{r},t) = (q_3/\hbar)\int_0^{\mathbf{r}} \vec{A}\cdot d\mathbf{r}' + S_{03}(\mathbf{r},t)$$

with

$$S_{02}(\mathbf{r},t) = \bar{\mathbf{k}}_2\cdot\mathbf{r} - \int_0^t dt'\big(V_{2eff}(\mathbf{r}+\bar{\mathbf{K}}_2(t'-t)) + U\rho(\mathbf{r}+\bar{\mathbf{K}}_2(t'-t))\big)$$

and

$$S_{03}(\mathbf{r},t) = \bar{\mathbf{k}}_3\cdot\mathbf{r} - \int_0^t dt'(V_{3eff}(\mathbf{r}+\bar{\mathbf{K}}_3(t'-t)) + U\rho(\mathbf{r}+\bar{\mathbf{K}}_3(t'-t))\big).$$

$\bar{\mathbf{K}}_2 = \hbar\bar{\mathbf{k}}_2/m = \hbar(\mathbf{k}_{p_1} - \mathbf{k}_{c_1})/m$ and $\bar{\mathbf{K}}_3 = \hbar\bar{\mathbf{k}}_3/m = \hbar(\mathbf{k}_{p_2} - \mathbf{k}_{c_2})/m$ are the corresponding recoil velocities. The velocity spread of the out-coupled matter fields can be obtained by $\vec{v}_j(\mathbf{r}) = \hbar\nabla_{\mathbf{r}}S_j(\mathbf{r},t)/m$ $(j = 2,3)$. It can be verified that the loop integration of the velocity yields

$$\oint_{z=0\in C_j} \vec{v}_j(\mathbf{r}')\cdot d\mathbf{r}' = \pm 2\pi\hbar l/m, \quad (j=2,3), \qquad (19)$$

where the sign of right-hand side of above equation takes + (for $j = 2$) or − (for $j = 3$). $z = 0 \in C_j$ means that the integration path C_j encircles the z-axis. The above results indicate that the orbital angular momentum of the input probe fields can be transferred into the out-coupled two-flavor atom lasers in corresponding vortex states (Fig. 1b). Practically, since the small ratio between the Rabi frequency of probe field and that of control field has a spatially distribution, strictly one can not obtain a vortex atom laser in usual three-level EIT technique [13]. Here, with EIT technique in a five-level M type system, the two-flavor atom lasers in a vortex state can be generated when the ratio between the Rabi frequencies of the first pair of probe and control fields ($\Omega_{p_1}^{(0)}$ and $\Omega_{c_1}^{(0)}$) equals that between the second pair ($\Omega_{p_2}^{(0)}$ and $\Omega_{c_2}^{(0)}$), while the spatially-dependent character of the ratio function itself is never restricted. The present system with effective ± charge may have very interesting applications. For example, if the different internal states represent different spin states of the atoms, and since the vortex states with different effective charges can be controlled independently by using external potentials, we can generate a spin current by controlling the atoms with different vortex states to move to opposite directions. This will be very useful for spintronics [25], which will be studied specifically in our next publication.

3 Discussion on nonclassical case

It is noteworthy that the above result can be extended to generate stationary continuous two-flavor vortex atom laser with non-classical atoms using the Fleischhauer-Gong technique [18,20]. For a brief discussion we consider a beam of five-level M type atoms moving in $+z$-direction interacts with two spatially varying control Stokes fields and two quantized probe fields with $\pm z$-directional orbital angular momentum $\pm l\hbar$, respectively. The control Stokes fields are taken to be much stronger than the probe ones. The Rabi-frequencies of the Stokes fields can be described by $\Omega_j(\mathbf{r},t) = \Omega_{0j}(\mathbf{r})e^{-i\omega_{s_j}(t-z/c_j)}$ with Ω_{0j} $(j = 1,2)$ being taken as real and c_j denoting the phase velocities projected onto the z-axis, the two quantized probe fields are characterized by the dimensionless positive frequency components $E_j^{(+)}(\mathbf{r},t) = \mathcal{E}_j(\mathbf{r},t)e^{-i\omega_{pj}(t-z/c)+il_j\phi}$ $(l_1 = -l_2 = l)$, and the ratio of the absolute value of Rabi frequencies between the first pair of probe and Stokes fields equals that between second pair, i.e. $g_1^2\langle\mathcal{E}_1^\dagger\mathcal{E}_1\rangle/\Omega_{01}^2 = g_2^2\langle\mathcal{E}_2^\dagger\mathcal{E}_2\rangle/\Omega_{02}^2$, where g_j is the coupling constant [7]. Similar to the former derivation in this paper, we may introduce the slowly-varying amplitudes

$$\Psi_1 = \Phi_1(\mathbf{r},t)e^{i(k_0z-\omega_0t)},$$
$$\Psi_2 = \Phi_2(\mathbf{r},t)e^{i[(k_0+k_{p_1}-k_{c_1})z-(\omega_0+\omega_{p_1}-\omega_{c_1})t]},$$
$$\Psi_3 = \Phi_3(\mathbf{r},t)e^{i[k_0+k_{p_2}-k_{c_2})z-(\omega_0+\omega_{p_2}-\omega_{c_2})]},$$
$$\Psi_4 = \Phi_4(\mathbf{r},t)e^{-i(k_0+k_{p_1})z-(\omega_0+\omega_{p_1})t}$$
$$\text{and} \quad \Psi_5 = \Phi_5(\mathbf{r},t)e^{-i(k_0+k_{p_2})z-(\omega_0+\omega_{p_2})t},$$

where $\hbar\omega_0 = \hbar^2k_0^2/2m$ is the corresponding kinetic energy in the mean velocity, k_{p_j} and k_{s_j} $(j = 1,2)$ are respectively the vector projections of the two probe and Stokes fields to the z axis. The atoms have a narrow velocity distribution around $v_0 = \hbar k_0/2m$ with $k_0 \gg |k_{p_j} - k_{s_j}|(j = 1,2)$ to minimize the effect of Doppler broadening [20]. All fields are assumed to be in resonance for the central velocity class.

We consider a stationary input of atoms in state $|1\rangle$, i.e., at the entrance region $|\Phi_1(\mathbf{r},t)|_{z=0} = (\rho(\mathbf{r}))^{1/2}|_{z=0}$. If the Rabi-frequencies Ω_{01} and Ω_{02} are sufficiently slowly, monotonically decreasing function of z but dependent on x and y and approach zero at the entrance $z = L$ and all the effective trap potentials are assumed to be zero [18], together with the former results, the output matter waves can then be obtained in the vortex state

$$\Phi_{2,3}(\mathbf{r},t)|_{z=L} = -\sqrt{\frac{c}{v_0}}\mathcal{E}_{1,2}\big(\mathbf{r},t-\tau_{2,3}(L)\big)|_{z=0}\exp\left[iS_{2,3}(\mathbf{r},t)\right], \qquad (20)$$

where $\tau_j(L) = \int_0^L dz'/V_g^{(j)}(z')$ and the group velocity

$$V_g^{(j)} = c\left(1 + \frac{g_j^2 n}{\Omega_{0j}^2}\frac{v_0}{c}\right) \Bigg/ \left(1 + \frac{g_j^2 n}{\Omega_{0j}^2}\right) \ (j = 2, 3)$$

with $n = \int \rho(\mathbf{r})dxdy$ is the total linear-density of the condensate [18,20]. The factor $\sqrt{c/v_0}$ accounts for the fact that the input light pulses propagate with velocity c and the output matter field propagates with v_0. In fact, the Raman process of present five-level system can be considered as two separate three-level Raman process, i.e. the process of atom field Φ_1 to Φ_2 using the first pair probe and control fields and the process of atom field Φ_1 to Φ_3 using the second pair probe and control fields. Then, the quantum states of the quantized probe field $\mathcal{E}_1$ can be transferred to the output atom field Φ_2 and the quantum states of probe field $\mathcal{E}_2$ can be transferred to the output atom field Φ_3, respectively. Particularly, when the entangled probe lights are used, we can obtain a pair of entangled vortex atom lasers. On other hand, in a certain extent the vortex states may be helpful to overcome the decoherence effect and other difficulties that restrict the atomic beam to propagate over a long distance. For this our result can be useful for quantum information processing based on the non-classical vortex atom lasers.

4 Conclusions

In conclusion we obtain a two-flavor atom laser in a vortex state via electromagnetically induced transparency (EIT) technique in a five-level M type system by using two probe lights with $\pm z$-directional orbital angular momentum $\pm l\hbar$, respectively. The key point of present technique is that if the the ratio between the Rabi frequencies of the first pair of probe and control fields equals that of the second pair, the equations of motion for the output atomic beams can be described as that of an effective two-flavor (oppositely charged) Bose condensate in an effective magnetic field, and then a vortex state of two-flavor atom laser will be obtained. Together with the original Fleischhauer-Gong scheme, the present result can be extended to generate continuous two-flavor vortex atom laser with non-classical atoms. Finally, it has other very interesting applications for the superposition state of different circulating vortex states labelled by the distinctive internal states. For example, if the different internal state represents different spin state of the atoms, and since the different vortex state can be controlled independently by using external potentials, we can generate a spin current by controlling the atoms with different vortex states to move to opposite directions. This will be very useful for spintronics, which will be studied specifically in our next work.

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